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Sensitivity Analysis of an Optimal Control Problem in Greenhouse Climate Management

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Optimal control systems are based on a performance measure to be optimised and a model description of the dynamic process to be controlled. When on-line implementation is considered, the performance of optimally controlled processes will depend on the accuracy of the model description used. Sensitivity analysis offers insight into the impact of uncertainty in the model parameters on the performance of the optimally controlled process. Additionally, sensitivity analysis may reveal the mechanisms underlying optimal process operation. This paper describes the methodology and results of a sensitivity analysis of an optimal control problem in greenhouse climate management. The methodology used, is based on variational arguments and requires a single solution of the optimal control problem, resulting in a computationally efficient technique. The example considered deals with economic optimal greenhouse climate management during the cultivation of a lettuce crop. The sensitivity analysis produced valuable insight into the performance sensitivity and operation of the controlled process. Both the model description of crop growth and production as well as the outside climate conditions have a strong impact on the performance. Humidity control plays a dominant role in economic optimal greenhouse climate management, emphasising the need for an accurate description of humidity effects on crop growth and production, either in terms of quantitative models or time-varying constraints on the humidity level in the greenhouse. Finally, the study revealed that the dynamic response times in the greenhouse climate are not limiting factors for economic optimal greenhouse climate control.

1. Introduction

The optimal control methodology is a powerful technique to facilitate the design and analysis of optimally controlled systems. Optimal control systems are based on a model description of the dynamic process to be controlled and are designed in such a way that a performance criterion is optimised with respect to the control action applied to the system (*e.g.* Pontryagin *et al.*, 1962). In practice, the structure as well as the parameter values of the model rarely coincide exactly with the real process. Since the control system is designed to be optimal with particular regard to the nominal structure and parameter values of the model used, it can be expected that the control system is sensitive to modelling errors which may reduce the performance of an optimal control system in practice. Therefore, sensitivity considerations are among the fundamental aspects of the synthesis and analysis of optimal control systems.

One way to assess performance sensitivity is to substitute one by one the original values of the model parameters by slightly perturbed values and to compute the new optimal control and corresponding value of the performance criterion. This, however, is a rather time consuming procedure. In this research, a first-order approach to the sensitivity analysis of open-loop optimal control problems was used as derived by Courtin and Rootenberg (1971) and Evers (1979, 1980). Using variational arguments, the methodology requires a single calculation of the open-loop optimal control and corresponding state and costate trajectories. These are then used to calculate a first-order approximation of the performance sensitivity, thus saving a considerable amount of computation time.

Notation

c model parameter
$c_{ai,ou}$ heat transmission coefficient through the greenhouse cover (6·1), $W\,m^{-2}\,{}^{\circ}C^{-1}$
$c_{cap,c}$ volumetric capacity of greenhouse air for carbon dioxide (4·1), m
$c_{cap,h}$ volumetric capacity of greenhouse air for humidity (4·1), m
$c_{cap,q}$ heat capacity of greenhouse air (30000), $J\,m^{-2}\,{}^{\circ}C^{-1}$
$c_{cap,q,v}$ heat capacity per volume unit of greenhouse air (1290), $J\,m^{-3}\,{}^{\circ}C^{-1}$
c_{co_2} costs of carbon dioxide (42×10^{-2}), $Hfl\,kg^{-1}$
$c_{co_{2},1}$ temperature effect on CO_2 diffusion in leaves ($5{\cdot}11\times10^{-6}$), $m\,s^{-1}\,{}^{\circ}C^{-2}$
$c_{co_{2},2}$ temperature effect on CO_2 diffusion in leaves ($2{\cdot}30\times10^{-4}$), $m\,s^{-1}\,{}^{\circ}C^{-1}$
$c_{co_{2},3}$ temperature effect on CO_2 diffusion in leaves ($6{\cdot}29\times10^{-4}$), $m\,s^{-1}$
c_{leak} leakage air exchange through greenhouse cover ($0{\cdot}75\times10^{-4}$), $m\,s^{-1}$
$c_{pl,d}$ effective canopy surface (53), $m^2\,kg^{-1}$
$c_{pri,1}$ parameter defining price of lettuce (1·8), $Hfl\,m^{-2}$
$c_{pri,2}$ parameter defining price of lettuce (16), $Hfl\,kg^{-1}$
c_q price of heating energy ($6{\cdot}35\times10^{-9}$), $Hfl\,J^{-1}$
c_R gas constant (8314), $J\,K^{-1}\,kmol^{-1}$
$c_{rad,phot}$ light use efficiency ($3{\cdot}55\times10^{-9}$), $kg\,J^{-1}$
$c_{rad,q}$ heat load coefficient due to solar radiation (0.2)
$c_{resp,d}$ respiration rate in terms of respired dry matter ($2{\cdot}65\times10^{-7}$), s^{-1}
$c_{resp,c}$ respiration rate in terms of produced carbon dioxide ($4{\cdot}87\times10^{-7}$), s^{-1}
$c_{T,abs}$ temperature in K at 0°C (273·15), K
$c_{v,pl,ai}$ canopy transpiration mass transfer coefficient ($3{\cdot}6\times10^{-3}$), $m\,s^{-1}$
$c_{v,1}$ parameter defining saturation water vapour pressure (9348), $J\,m^{-3}$
$c_{v,2}$ parameter defining saturation water vapour pressure (17·4)
$c_{v,3}$ parameter defining saturation water vapour pressure (239), °C
$c_{v,4}$ parameter defining saturation water vapour pressure (10998), $J\,m^{-3}$
c_{V_c} perturbation parameter on carbon dioxide concentration outside greenhouse (1)
$c_{V_{rad}}$ perturbation parameter on solar radiation outside greenhouse (1)
c_{V_h} perturbation parameter on humidity outside greenhouse (1)
c_{V_T} perturbation parameter on temperature outside greenhouse (1)
$c_{\alpha\beta}$ yield factor (0·544)
c_σ weighting factor in penalty function, $Hfl\,m^{-2}\,s^{-1}$
c_Γ carbon dioxide compensation point ($5{\cdot}2\times10^{-5}$), $kg\,m^{-3}$
H Hamiltonian, $Hfl\,m^{-2}\,s^{-1}$
J performance measure, $Hfl\,m^{-2}$
m number of model parameters
n number of state variables
i, j, k iteration numbers
p penalty
p_T penalty for constraint violations by greenhouse air temperature, $Hfl\,m^{-2}\,s^{-1}$
p_c penalty for constraint violations by carbon dioxide concentration, $Hfl\,m^{-2}\,s^{-1}$
p_h penalty for constraint violations by humidity, $Hfl\,m^{-2}\,s^{-1}$
$Q_{vent,q}$ energy exchange by ventilation and transmission through the cover, $W\,m^{-2}$
$Q_{rad,q}$ heat load by solar radiation, $W\,m^{-2}$
R_{Xh} relative humidity
t time
t_b start time of optimisation interval
t_f end time of optimisation interval
u control input
U_c supply rate of carbon dioxide, $kg\,m^{-2}\,s^{-1}$
U_q energy supply by the heating system, $W\,m^{-2}$
U_v ventilation rate, $m\,s^{-1}$
V_c carbon dioxide concentration outside the greenhouse, $kg\,m^{-3}$
V_h outdoor humidity concentration, $kg\,m^{-3}$
V_{rad} solar radiation outside the greenhouse, $W\,m^{-2}$
V_T outdoor temperature, °C
x state variable
X_c carbon dioxide concentration in greenhouse, $kg\,m^{-3}$
X_d crop dry weight, $kg\,m^{-2}$
X_h humidity concentration in greenhouse, $kg\,m^{-3}$
X_T air temperature in the greenhouse, °C
λ costate
λ_d costate of crop dry weight, $Hfl\,kg^{-1}$
λ_c costate of carbon dioxide concentration, $Hfl\,m\,kg^{-1}$
λ_h costate of humidity concentration, $Hfl\,m\,kg^{-1}$
λ_T costate of air temperature, $Hfl\,m^{-2}\,{}^{\circ}C^{-1}$
$\varphi_{phot,c}$ gross canopy photosynthesis rate, $kg\,m^{-2}\,s^{-1}$
$\varphi_{vent,c}$ mass exchange of carbon dioxide through the vents, $kg\,m^{-2}\,s^{-1}$

$\varphi_{vent,h}$	mass exchange of humidity through the vents, $kg\,m^{-2}\,s^{-1}$	*sat*	saturation level
$\varphi_{transp,h}$	canopy transpiration, $kg\,m^{-2}\,s^{-1}$	*Superscripts*	
		$*$	optimal value
Subscripts		$\circ$	nominal value
min	lower bound on control or state variable		
max	upper bound on control or state variable		

The optimal control problem considered in this paper, deals with economic optimal operation of the climate conditioning equipment in a greenhouse. To improve the economic performance of greenhouse crop production, in this approach, greenhouse climate control is based on an explicit trade-off between costs of operating the climate conditioning equipment and the economic return of the crop production process. This optimal control approach has received considerable attention in the agricultural engineering society (*e.g.* Chalabi, 1992; Hwang, 1993; Van Henten, 1994; Seginer & Ioslovich, 1998; Tap, 2000). However, parameter sensitivity issues have hardly been investigated in this field of research. Chalabi and Bailey (1991) as well as Van Henten and Van Straten (1994) performed sensitivity analyses of dynamic models of the greenhouse climate and crop growth of lettuce, respectively. The results, though being of interest from a modelling point of view, only allow for qualitative conclusions about the impact of model uncertainty on the performance of an optimal control system based on such a model. It is the objective of this paper to directly address the performance sensitivity of an economic optimal greenhouse climate control problem with respect to small perturbations in the model parameters.

2. Materials and methods

2.1. *Definition of the optimal control problem*

The research reported in this paper focussed on economic optimal greenhouse climate management during the production of a lettuce crop. The objective was to maximise the net economic return of the crop production process. The net economic return J of the lettuce production process, expressed in Dutch guilders per square metre greenhouse ($Hfl\,m^{-2}$), was described by the equation:

$$J = \left(c_{pri,1} + c_{pri,2}X_d(t_f)\right) - \int_{t_b}^{t_f} \left(c_q U_q(t) + c_{co_2} U_c(t)\right) \mathrm{d}t \tag{1}$$

where $c_{pri,1} + c_{pri,2}X_d(t_f)$ in $Hfl\,m^{-2}$ is the gross income obtained at harvest time t_f when selling the harvested product at the auction, and $c_q U_q(t) + c_{co_2} U_c(t)$ is the running costs of the climate conditioning equipment in $Hfl\,m^{-2}\,s^{-1}$. Analysis of the auction price of lettuce in the period 1985–1990 revealed this linear relationship, parameterised by $c_{pri,1}$ in $Hfl\,m^{-2}$ and $c_{pri,2}$ in $Hfl\,kg^{-1}$, between the auction price and the harvest weight of lettuce X_d in $kg\,m^{-2}$ (Van Henten, 1994). The running costs of the climate conditioning equipment were assumed to be linearly related with the amount of energy U_q in $W\,m^{-2}$ and the amount of carbon dioxide U_c in $kg\,m^{-2}\,s^{-1}$, put into the system. These running costs were parameterised by the energy price c_q in $Hfl\,J^{-1}$ and the price of carbon dioxide c_{co_2} in $Hfl\,kg^{-1}$, respectively. It was assumed that no costs were associated with natural ventilation used for cooling and dehumidification. The contribution of the electrical equipment used for greenhouse climate conditioning, such as pumps and valves, to the operating costs was ignored. Furthermore, it was assumed that other production factors, such as the nutrient and water supply, screening and those not directly related to greenhouse climate control, such as labour input, pest and disease control, do not affect the control strategies. Consequently, they are not included in the performance criterion. The running costs were integrated over the whole growing period starting at the planting date t_b and ending at harvest time t_f. Then, subtraction of the integrated operating costs from the gross income yielded the net economic return of the crop production process to be optimised.

The crop production process was described by a four-state variable dynamic model. In this analysis both the greenhouse climate dynamics as well as the crop growth dynamics were considered. The model described the evolution in time of the dry matter content of the crop X_d in $kg\,m^{-2}$, the carbon dioxide concentration in the greenhouse X_c in $kg\,m^{-3}$, the air temperature in the greenhouse X_T in °C and the humidity content of the greenhouse air X_h in $kg\,m^{-3}$, with the equations:

$$\frac{\mathrm{d}X_d}{\mathrm{d}t} = c_{\alpha\beta}\varphi_{phot,c} - c_{resp,d}X_d 2^{(0\cdot 1X_T - 2\cdot 5)} \tag{2}$$

where: $c_{\alpha\beta}$ is a yield factor, $\varphi_{phot,c}$ is the gross canopy photosynthesis rate in $\mathrm{kg\,m^{-2}\,s^{-1}}$, $c_{resp,d}$ in $\mathrm{s^{-1}}$ is the respiration rate expressed in terms of the amount of respired dry matter and X_T is the air temperature in the greenhouse in °C,

$$\frac{\mathrm{d}X_c}{\mathrm{d}t} = \frac{1}{c_{cap,c}}\left[-\varphi_{phot,c} + c_{resp,c}X_d 2^{(0.1X_T-2.5)} + U_c - \varphi_{vent,c}\right] \tag{3}$$

where: $c_{cap,c}$ is the volumetric carbon dioxide capacity of the greenhouse air in $\mathrm{m^3\,m^{-2}}$, $c_{resp,c}$ in $\mathrm{s^{-1}}$ is the respiration coefficient expressed in terms of the amount of carbon dioxide produced, U_c is the supply rate of carbon dioxide in $\mathrm{kg\,m^{-2}\,s^{-1}}$ and $\varphi_{vent,c}$ is the mass exchange of carbon dioxide through the vents in $\mathrm{kg\,m^{-2}\,s^{-1}}$,

$$\frac{\mathrm{d}X_T}{\mathrm{d}t} = \frac{1}{c_{cap,q}}\left[U_q - Q_{vent,q} + Q_{rad,q}\right] \tag{4}$$

where: $c_{cap,q}$ is the heat capacity of the greenhouse air in $\mathrm{J\,m^{-2}\,{}^\circ C^{-1}}$, U_q is the energy supply by the heating system in $\mathrm{W\,m^{-2}}$, $Q_{vent,q}$ is the energy exchange with the outdoor air by means of ventilation and transmission through the cover in $\mathrm{W\,m^{-2}}$ and $Q_{rad,q}$ is the heat load by solar radiation in $\mathrm{W\,m^{-2}}$,

$$\frac{\mathrm{d}X_h}{\mathrm{d}t} = \frac{1}{c_{cap,h}}\left[\varphi_{transp,h} - \varphi_{vent,h}\right] \tag{5}$$

where: $c_{cap,h}$ is the volumetric water vapour capacity of the greenhouse air in $\mathrm{m^3\,m^{-2}}$, $\varphi_{transp,h}$ is the canopy transpiration in $\mathrm{kg\,m^{-2}\,s^{-1}}$ and $\varphi_{vent,h}$ is the mass exchange of water vapour through the vents in $\mathrm{kg\,m^{-2}\,s^{-1}}$.

The gross photosynthesis rate $\varphi_{phot,c}$ in kg $\mathrm{m^{-2}}$ $\mathrm{s^{-1}}$, is described by

$$\varphi_{phot,c} = \left(1 - \mathrm{e}^{-c_{pl,d}X_d}\right) \frac{c_{rad,phot}V_{rad}\left(-c_{co_{2,1}}X_T^2 + c_{co_{2,2}}X_T - c_{co_{2,3}}\right)(X_c - c_\Gamma)}{c_{rad,phot}V_{rad} + \left(-c_{co_{2,1}}X_T^2 + c_{co_{2,2}}X_T - c_{co_{2,3}}\right)(X_c - c_\Gamma)} \tag{6}$$

where: $c_{pl,d}$ is the effective canopy surface in $\mathrm{m^2\,kg^{-1}}$, $c_{rad,phot}$ is the light use efficiency in $\mathrm{kg\,J^{-1}}$, V_{rad} is the solar radiation outside the greenhouse in $\mathrm{W\,m^{-2}}$, $c_{co_{2,1}}$ in $\mathrm{m\,s^{-1}\,{}^\circ C^{-2}}$, $c_{co_{2,2}}$ in $\mathrm{m\,s^{-1}\,{}^\circ C^{-1}}$ and $c_{co_{2,3}}$ in $\mathrm{m\,s^{-1}}$ parameterise the temperature influence on gross canopy photosynthesis, c_Γ is the carbon dioxide compensation point in $\mathrm{kg\,m^{-3}}$. The mass transfer of carbon dioxide due to ventilation and leakage $\varphi_{vent,c}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is defined by

$$\varphi_{vent,c} = (U_v + c_{leak})(X_c - V_c) \tag{7}$$

where: U_v is the ventilation rate through the vents in $\mathrm{m\,s^{-1}}$, c_{leak} is the leakage through the cover in $\mathrm{m\,s^{-1}}$ and V_c is the carbon dioxide concentration outside the greenhouse in $\mathrm{kg\,m^{-3}}$. The energy transfer between the indoor environment and the outdoor environment due to ventilation and transmission $Q_{vent,q}$ in $\mathrm{W\,m^{-2}}$, is covered by the equation

$$Q_{vent,q} = \left(c_{cap,q,v}U_v + c_{ai,ou}\right)(X_T - V_T) \tag{8}$$

in which $c_{cap,q,v}$ is the heat capacity per volume unit of greenhouse air in $\mathrm{J\,m^{-3}\,{}^\circ C^{-1}}$, $c_{ai,ou}$ in $\mathrm{W\,m^{-2}\,{}^\circ C^{-1}}$ parameterises the transfer of sensible heat through the cover, V_T in °C stands for the outside air temperature. The energy input to the greenhouse system by solar radiation $Q_{rad,q}$ in $\mathrm{W\,m^{-2}}$, is described by:

$$Q_{rad,q} = c_{rad,q}V_{rad} \tag{9}$$

where: $c_{rad,q}$ is the heat load coefficient due to solar radiation. Canopy transpiration $\varphi_{transp,h}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is governed by the equation

$$\varphi_{transp,h} = \left(1 - \mathrm{e}^{-c_{pl,d}X_d}\right)c_{v,pl,ai} \left(\frac{c_{v,1}}{c_R\left(X_T + c_{T,abs}\right)}\mathrm{e}^{c_{v,2}X_T/(X_T+c_{v,3})} - X_h\right) \tag{10}$$

in which the term $c_{v,1}\mathrm{e}^{c_{v,2}X_T/X_T+c_{v,3}}/c_R(X_T + c_{T,abs})$ in $\mathrm{kg\,m^{-3}}$ represents the saturated water vapour content at canopy temperature X_T, $c_{v,pl,ai}$ is the mass transfer coefficient in $\mathrm{m\,s^{-1}}$, $c_{v,1}$ in $\mathrm{J\,m^{-3}}$, $c_{v,2}$ and $c_{v,3}$ in °C parameterise the saturation water vapour pressure, c_R is the gas constant in $\mathrm{J\,K^{-1}\,kmol^{-1}}$ and $c_{T,abs}$ is the temperature in K at 0°C. The mass transfer of water vapour by means of ventilation $\varphi_{vent,h}$ in kg $\mathrm{m^{-2}}$ $\mathrm{s^{-1}}$, is described by

$$\varphi_{vent,h} = (U_v + c_{leak})(X_h - V_h) \tag{11}$$

in which V_h in $\mathrm{kg\,m^{-3}}$ is the humidity concentration outside the greenhouse.

The model, though being of rather simple structure was found to describe measured data rather well. For a more detailed description and verification of this model is referred to Van Henten (1994).

Physical limitations on the control inputs U_c, U_q and U_v, were represented by the linear inequality constraints $X_{c,min} \le U_c \le U_{c,max}$, $X_{q,min} \le U_q \le U_{q,max}$, $X_{v,min} \le U_v \le U_{v,max}$, respectively. Bounds were also imposed on the temperature in the greenhouse X_T, the carbon dioxide concentration X_c and the humidity level X_h, to prevent the control system from driving the process into unfavourable conditions for crop growth and development. These bounds were represented by the linear inequality constraints $X_{T,min} \le X_T \le X_{T,max}$, $X_{c,min} \le X_c \le X_{c,max}$ and $X_{h,min} \le X_h \le X_{h,max}$. In fact, these bounds represented the limitations of the rather simple crop growth model used in this research, since the adverse effect of unfavourable climate conditions on crop growth and development should have been covered by the dynamic crop growth model. In the example considered, bounds were imposed on the relative humidity instead of the absolute humidity. This required the transformation $X_{h,min} = \frac{R_{Xh,min}}{100} X_{h,sat}(X_T)$ and $X_{x,max} = \frac{R_{Xh,max}}{100} X_{h,sat}(X_T)$ with $R_{Xh,min}$ and $R_{Xh,max}$ being the lower and upper bound on the relative humidity, respectively and the saturation water vapour pressure $X_{h,sat}$ in kg m^{-3}, is a function of the temperature X_T:

$$X_{h,sat} = \frac{c_{v,4}}{c_R(X_T + c_{T,abs})} e^{c_{v,2} X_T/(X_T + c_{v,3})} \tag{12}$$

where $c_{v,4}$ in J m^{-3} parameterises the saturation water vapour pressure together with the parameters $c_{v,2}$ and $c_{v,3}$.

To deal with the inequality constraints on the state variables, following Pierre (1969), the performance criterion of Eqn (1) was extended with penalty functions $p(t)$ in Hfl m^{-2} s^{-1}, having the general form

$$p(t) = c_\sigma \left[\frac{2X(t) - X_{min} - X_{max}}{X_{min} - X_{max}} \right]^{2k} \tag{13}$$

where: c_σ Hfl m^{-2} s^{-1} is a weighting factor, X is a state variable, X_{min} and X_{max} are the lower and the upper bound put on the state variable, respectively, and the exponent k forces the penalty function to attain values near zero between the bounds and very steep slopes close to the bounds when $k = 1, 2; .., \infty$. In this way, the controlled system was prevented from traversing the bounds. To guarantee consistence in the units used, the penalty was expressed in Hfl m^{-2} s^{-1}. In a way, this is a slightly artificial construction, though one may argue that by modifying the weighting parameter c_σ the grower is able to express his attitude towards taking risks, when the health of the crop is considered. After adding penalties for violations of the constraints on the temperature, carbon dioxide concentration and humidity, p_T, p_c and p_h respectively, the resulting performance measure had the following form

$$J = \left(c_{pri,1} + c_{pri,2} X_d(t_f)\right) - \int_{t_b}^{t_f} \left(c_q U_q(t) + c_{co_2} U_c(t) + p_c(t) + p_T(t) + p_h(t)\right) \mathrm{d}t \tag{14}$$

With the preliminaries presented above, the optimal control problem was defined as to find optimal control strategies for the control variables U_c, U_q and U_v over the time-interval $t \in [t_b, t_f]$, maximising the performance criterion of Eqn (14), subject to the differential equation constraints of Eqns (2)–(5) and the linear inequality constraints on the controlled variables.

2.2. *Solution of the optimal control problem*

The optimal control problem was solved using measured data of the outside climatic conditions obtained during a greenhouse experiment in early 1992 (Van Henten, 1994). These data consisted of 2-min measurements of the external inputs, *i.e.* the outdoor temperature, humidity, carbon dioxide concentration and solar radiation. A growing period of only 50 days was considered. An iterative scheme based on the Maximum Principle of Pontryagin (Pontryagin *et al.*, 1962) was used to find the optimal control trajectories (Kirk, 1970). A crucial step in the solution of the optimal control problem is the derivation of the Hamiltonian (*e.g.* Kirk, 1970). For the example considered, the Hamiltonian H has the following form:

$$\begin{aligned} H = & -c_{co_2} U_c - c_q U_q + \lambda_d \left\{ c_{\alpha\beta} \varphi_{phot,c} - c_{resp,d} X_d 2^{(0.1 X_T - 2.5)} \right\} \\ & + \lambda_c \left\{ \frac{1}{c_{cap,c}} \left[-\varphi_{phot,c} + c_{resp,c} X_d 2^{(0.1 X_T - 2.5)} + U_c - \varphi_{vent,c} \right] \right\} \\ & + \lambda_T \left\{ \frac{1}{c_{cap,q}} \left[U_q - Q_{vent,q} + Q_{rad,q} \right] \right\} \\ & + \lambda_h \left\{ \frac{1}{c_{cap,h}} \left[\varphi_{transp,h} - \varphi_{vent,h} \right] \right\} - p_c - p_T - p_h \end{aligned} \tag{15}$$

in which λ_d in Hfl kg^{-1}, λ_c in Hfl m kg^{-1}, λ_T in Hfl m^{-2} °C^{-1} and λ_h in Hfl m kg^{-1} are the so-called adjoint variables or costates related to the state variables X_d, X_c, X_T and X_h. The dynamics of the costates are described by the equation:

$$-\dot{\lambda} = \frac{\partial H}{\partial x} \tag{16}$$

where λ is the costate and x is the state variable. The costates express the marginal value of a change in the associated state variables. If a costate is positive, an increment of the associated state variable will have a positive effect on the final net economic return, and vice versa. The Hamiltonian can be seen as a momentary profit rate in which current costs are balanced against future revenues. In this way the Hamiltonian is a great source of information for interpretation of the results of the sensitivity analysis in the next section.

The Maximum Principle of Pontryagin asserts that to maximise the performance criterion in Eqn (14) it is sufficient to maximise the Hamiltonian at all time instants in the optimisation interval, *i.e.*:

$$H(x^*, u^*, \lambda^*, t) \geq H(x^*, u, \lambda^*, t) \tag{17}$$

in which x^*, u^* and λ^* are the optimal values of the states, control inputs and costates.

In the iterative solution, the state and costate equations were simulated in double-precision with an integration time step of half a minute using a fourth-order

Runge–Kutta algorithm described by Press *et al.* (1986). A modified steepest ascent algorithm exploiting the gradient information $\partial H/\partial u$, was used for the iterative solution of the optimal control problem (Kirk, 1970; Van Henten, 1994).

2.3. *First-order sensitivity analysis*

Using variational arguments, first-order approximations of the performance sensitivity were derived by Courtin and Rootenberg (1971) and Evers (1979, 1980). The performance sensitivity with respect to the values of the state variables at the beginning of the growing period $x(t_b)$, equals

$$\frac{\partial J}{\partial x_i(t_b)} = \lambda_i^*(t_b), \quad i = 1, \ldots, n \tag{18}$$

where $\lambda_i^*(t_b)$ is the optimal value of the costate at the start time t_b and n is the number of state variables. The performance sensitivity with respect to the model parameters c is

$$\frac{\partial J}{\partial c_j} = \int_{t_b}^{t_f} \frac{\partial H}{\partial c_j}\left(x^*, u^*, c^0, t\right) \mathrm{d}t, \quad j = 1, \ldots, m \tag{19}$$

in which x^* and u^* are the optimal state and control trajectories, respectively, c^0 denotes the nominal value of the model parameter and m represents the total number of model parameters.

The calculation of the first-order sensitivity contained two steps. First of all, the open-loop optimal control problem was solved. Then secondly, the effect of small perturbations of the model parameters on the performance measure was evaluated using the above-mentioned first-order measure of the performance sensitivity with respect to parameter perturbations. For the model parameters this first-order measure was obtained by integrating the partial derivatives of the Hamiltonian with respect to the model parameters over the whole optimisation interval. In the actual computation, these partial derivatives can be calculated analytically or numerically with a central difference approximation (*e.g.* Gill *et al.*, 1981). In this research, analytical derivatives were obtained and implemented in the simulation software based on FORTRAN. For the initial conditions of the state variables, the performance sensitivity was determined by the value of the associated costates at the starting time t_b.

In order to compare the impact of perturbations in the different model parameters and the initial conditions of the state variables on the system performance, it was considered to be more convenient to express the sensitivity as the fractional change in the performance criterion as a result of the fractional change in the parameter value, *i.e.* a relative sensitivity criterion. For every state variable and model parameter, the relative sensitivity measure was defined as

$$\frac{\partial J}{\partial x_i(t_b)} \frac{x_i^0(t_b)}{J^*}, \; i = 1, \ldots, n \quad \text{and} \quad \frac{\partial J \, c_j^0}{\partial c_j J^*}, \quad j = 1, \ldots, m. \tag{20}$$

By doing so, the interpretation of the results became straightforward. A relative sensitivity measure larger (less) than zero indicated that a small positive perturbation in the parameter resulted in an increase (decrease) of the value of the performance criterion. To be more precise, a value of the relative sensitivity measure of 1, indicated that a parameter change of 1% should result in a 1% change of the value of the performance criterion. For a relative performance sensitivity measure having a value larger or smaller than unity, the interpretation changed accordingly. As the first-order sensitivity analysis was based on a first-order Taylor series approximation of the change in performance due to a change in a parameter, the validity of the previous interpretation was limited to small parameter variations only. Still, the relative performance sensitivity measure should provide valuable insight into the contribution of certain model parameters to the control strategies calculated.

Besides, the performance sensitivity with respect to variations in the initial conditions and model parameters, the performance sensitivity with respect to small perturbations in the external inputs was evaluated. This was accomplished by multiplying each external input with a time invariant parameter, *i.e.* c_{V_c}, $c_{V_{rad}}$, c_{V_h} and c_{V_T}, having a nominal value of 1. Clearly, the effect of this perturbation parameter was that throughout the whole optimisation interval the external inputs are perturbed by the same amount. This may not sound fully realistic but it should give an impression of the relative importance of the external inputs on the performance of the optimal controlled process. In the sensitivity analysis, these perturbation parameters were treated as normal model parameters.

3. Results and discussion

The results of the sensitivity analysis are presented in Table 1. These results show that the constraint on the relative humidity seems to play a primary role in the performance of the optimal control problem. The importance of the humidity state constraint is indicated by the large sensitivity measure of c_R, $c_{T,abs}$, $c_{v,2}$, $c_{v,3}$ and $c_{v,4}$, parameterising the saturation water vapour pressure used in the definition of the constraint on the relative humidity. Although these parameters are also involved in the description of the canopy transpiration, their effect on the performance of the control strategies through the canopy transpiration seems to be of secondary importance. This can be inferred from the sign of the sensitivity measure. Taking c_R as an example, this can be seen as follows. An increment in the value of c_R results in a reduction of the saturation water vapour pressure. When the humidity constraint is encountered during daytime, the costate λ_h takes large negative values as can be seen in *Fig. 1*. This indicates the required reduction of the humidity in the greenhouse. Focusing on the humidity balance in the Hamiltonian equation, it can be seen that given $\lambda_h < 0$ and assuming the saturation water vapour pressure to be larger than the absolute humidity level in the greenhouse, any reduction in the water vapour pressure would result in a larger value of the Hamiltonian, thus suggesting a positive effect of an increment in c_R on the performance measure. The sign of the sensitivity measure, however, is negative. Therefore another effect of c_R is dominating. Close to the humidity constraint, the partial derivative of the penalty function takes very large positive values. Then, any reduction of the saturation water vapour pressure will result in an increasing penalty, thus yielding the negative effect on the performance measure observed in the sensitivity analysis. Apparently, the penalty function related to the humidity constraint dominates the Hamiltonian, thus emphasising the importance of an accurate definition of the humidity constraint in optimal greenhouse climate control.

Table 1
The relative sensitivity of the model parameter in decreasing order of their absolute values

Parameter	*Relative sensitivity*	*Parameter*	*Relative sensitivity*
$c_{v,2}$	5·4178	$c_{pl,d}$	−0·1542
$c_{v,4}$	4·5173	$c_{co_{2,3}}$	−0·1399
$c_{v,3}$	−3·9328	c_{leak}	−0·1116
c_{V_c}	1·6637	$c_{cap,q,v}$	−0·0958
$c_{\alpha\beta}$	1·7807	c_{V_T}	0·0963
$c_{V_{rad}}$	1·2627	$c_{cap,h}$	0·0919
$c_{rad,phot}$	1·1783	$X_d(t_b)$	0·0600
c_{V_h}	−1·0804	$c_{cap,c}$	−0·0500
c_R	−1·0800	$c_{resp,c}$	0·0148
$c_{T,abs}$	−1·0349	c_Γ	−0·0123
$c_{co_{2,2}}$	0·8742	$c_{cap,q}$	−0·0095
$c_{co_{2,1}}$	−0·3668	$c_{v,1}$	−0·0064
c_q	−0·3617	$X_t(t_b)$	0·0007
$c_{ai,ou}$	−0·3418	$c_{rad,q}$	0·0004
$c_{v,pl,ai}$	−0·3313	$X_h(t_b)$	−0·0003
$c_{resp,d}$	−0·2772	$X_c(t_b)$	0·0001
c_{co_2}	−0·1672		

As the constraint on the relative humidity is of such great importance in the control strategies, an accurate description of the humidity balance in the greenhouse, including processes like canopy transpiration, seems required. This is confirmed by the relative large performance sensitivity of parameter $c_{v,pl,ai}$ expressing the mass transfer coefficient for evaporative water vapour transport from the leaves to the ambient air. Under equal circumstances, a small positive increment in this parameter will result in a higher canopy transpiration and, consequently, it will result in an earlier conflict with the humidity constraint. The accompanying increment in the value of the penalty results in the negative sensitivity measure in Table 1.

In this example of lettuce cultivation, the gross economic return is determined by the dry matter production. Table 1 shows that most of the crop-related parameters more or less affect the performance of the control strategies. The sensitivity analysis of a lettuce growth model reported by Van Henten and Van Straten (1994) revealed the importance of parameters such as $c_{\alpha\beta}$, $c_{pl,d}$, $c_{rad,phot}$ and $c_{co_{2,2}}$. Consequently, in the present study a significant performance sensitivity for perturbations in these parameters was expected as well. Apart from the parameter $c_{pl,d}$, Table 1 shows the expected relatively large performance sensitivity for these parameters, thus emphasising the fact that for optimal greenhouse climate control their accurate parameterisation is required. The performance sensitivity for parameter $c_{pl,d}$, however, is much less distinct than was expected. The reason for this is the fact that $c_{pl,d}$ is also involved in the humidity balance of the greenhouse in which it describes the effective surface of the canopy. Before canopy closure, any increment of the canopy transpiration will result in more frequent conflicts with the humidity constraints. This has a negative effect on the performance measure, thus partly outweighing the

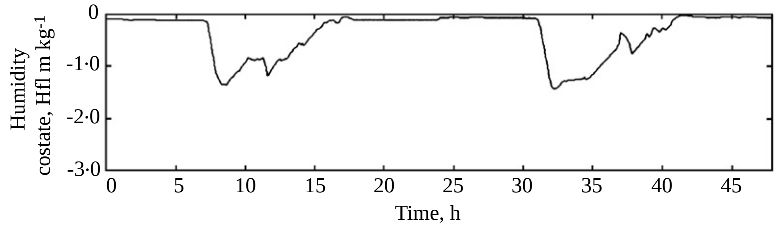

Fig. 1. The costate trajectory of the humidity during 2 days of the 50 days optimisation interval.

positive effects of early canopy closure on dry matter production of the canopy.

Since the important role of the humidity has been discussed in some detail above, further analysis is focused on the parameters in the energy and carbon dioxide balances. Parameters that most clearly affect the dynamic behaviour of the carbon dioxide concentration and air temperature in the greenhouse are the mass and heat capacities $c_{cap,c}$ and $c_{cap,q}$ respectively. Table 1 reveals that the effect of a perturbation in their values on the performance is relatively small. The heat and mass capacities of the greenhouse air determine to a large extent the dynamic rate with which the greenhouse climate can be modified: a larger capacity will result in a longer response time. The small performance sensitivity suggests that the greenhouse climate system is fast enough to deal with fast fluctuations in the external inputs in an economic optimal fashion. This can be seen as follows. If large benefits can be obtained by rapid modifications of the greenhouse climate anticipating rapid fluctuations in the external inputs such as the solar radiation, any decrease in response time will contribute to a significant improvement of the economic performance. Then, a pronounced performance sensitivity with respect to these parameters is expected. Compared with for instance crop growth-related parameters, however, $c_{cap,c}$ and $c_{cap,q}$have a small impact on the performance. Apparently, the response time of the greenhouse climate is not a limiting factor in the economic optimal control of the crop production process. Or alternatively, the relatively small performance sensitivity to changes in the heat and mass capacity of the greenhouse air suggest that in economic optimal greenhouse climate control very fast modifications of the greenhouse climate do not contribute much to an improvement of the economic performance.

These observations are in line with the results of the sensitivity analysis of the two-state variable crop growth model done by Van Henten and Van Straten (1994). Then it was found that crop growth is much more sensitive to changes in the long-term average of, for instance, the carbon dioxide concentration than to rapid fluctuations. Since in this example the performance of optimal greenhouse climate control is largely determined by the dry matter production, this would suggest a large performance sensitivity for parameters affecting the average indoor climate. In the present sensitivity analysis, this is confirmed by the significant sensitivity of the performance measure to a change in the carbon dioxide concentration in the outside air induced by c_{V_c}. Clearly, such a change does affect the long-term average carbon dioxide concentration in the greenhouse air, but not so much its dynamic rate of change.

In the sensitivity analysis of Van Henten and Van Straten (1994), it was concluded that during the day lettuce growth is not strongly influenced by the air temperature in the greenhouse. Due to this relatively low-temperature sensitivity of crop growth and the comparably high heating costs, the greenhouse air is rarely heated during the day. Still, parameter $c_{ai,ou}$, describing the energy losses to the outside air by means of transmission through the greenhouse cover and natural ventilation through the windows, shows a high performance sensitivity. During the major part of the growing period, heating energy is supplied to the greenhouse at night to satisfy the minimum temperature constraint. This minimum temperature constraint to a large extent determines the total energy consumption. Any reduction in the energy loss to the outside air will result in a reduction of the energy consumption required for heating the greenhouse. This results in a negative performance sensitivity. As the greenhouse climate is not exposed to rapid changes in the outside conditions during nighttime, economic optimal control does not require extremely fast modifications of the greenhouse air temperature. This explains the relatively low performance sensitivity of the heat capacity $c_{cap,q}$.

The very large positive performance sensitivity for perturbations in the solar radiation and outside carbon dioxide concentration is explained by the large sensitivity of crop growth for these climatic conditions. The large negative sensitivity for an increase in the outside humidity is due to the constraint on the relative humidity which will then be more difficult to satisfy. The large performance sensitivities emphasise the need for accurate assessment, *i.e.* prediction and measurement, of these outside climatic conditions.

The total operating costs of the climate conditioning equipment ($\pm 1{\cdot}00\,\mathrm{Hfl\,m^{-2}}$) is relatively small compared with the gross economic return of the crop production ($\pm 5{\cdot}25\,\mathrm{Hfl\,m^{-2}}$). Therefore a relatively small performance sensitivity for the operating costs expressed by the parameters c_{co_2} and c_q is found. Since in the simulations the overall heating costs exceeded the costs for carbon dioxide supply, the performance sensitivity for c_{co_2} is smaller than for c_q.

For all state variables the performance sensitivity to a perturbation in their initial conditions is relatively small. For the greenhouse climate variables this is explained by the fact that due to the fast system dynamics a small perturbation in the greenhouse climate has a very short lifetime which does not affect the performance of the system significantly. The low sensitivity of the initial dry weight of the crop, however, is unexpected. The start weight of the crop would be expected to have a significant positive effect on the final economic return. In this example, this still will be the case. In the optimisation, however, rapid crop growth also has an adverse effect on the performance because the size of the crop has a positive influence on canopy transpiration and thus will result in humidity constraint violations. In this example, this negative effect partly outweighed the positive impact of a large start weight.

Finally, it is interesting to note that some of the insights obtained in this research are in line with experiences from horticultural practice. As in the example in this research, in Dutch horticultural practice humidity considerations play a dominant role in ventilation control. In practice, abundant ventilation is used as a risk-aversion strategy against physiological disorder and diseases, but the underlying mechanisms are not yet very well understood. This research clearly emphasises that a more prudent use of ventilation for humidity control can have a significant positive effect on the economic performance of greenhouse crop production. The last few years this notion has been gaining interest in horticultural practice as well. Another example is the thermal heat loss through the greenhouse cover, in this research represented by the parameter $c_{ai,ou}$. The high performance sensitivity confirms that growers rightly choose to reduce energy losses through the greenhouse cover by means of thermal isolation.

4. Conclusions

With respect to the methodology used in this research the following conclusions are drawn. First of all, it was found that the first-order sensitivity analysis is a simple and straightforward way to obtain deeper insight into the operation of the optimal control problem and the relative importance of the model parameters, the initial conditions of the state variables and the external inputs, without having to go through extensive recalculations of the optimal control strategies. Secondly, the present study confirmed that a sensitivity analysis of the model to parameter variations and a sensitivity analysis of an optimal control problem including the same model might yield different results. Though the process model has an undisputed role in optimal control, it is the balancing of various objectives such as costs, revenues and penalties that determine the optimal control strategies. Finally, the intermediate variables in the solution of the optimal control problem such as the Hamiltonian and the costate trajectories, were found to be instrumental for a better understanding of the role of the model and model parameters in the determination of the optimal control strategies.

Application of the sensitivity analysis to an optimal control problem in greenhouse crop production, led to the following conclusions.

(1) The constraint on the humidity strongly influences the performance of optimal greenhouse climate management. Since these constraints were included as a first step to deal with adverse effects of high relative humidity on the quality of the crop, future research on greenhouse climate management should focus on a proper assessment of these effects in terms of quantitative models or modified climate constraints.

(2) In optimal greenhouse climate management, the dynamics of crop growth play a dominant role and require accurate models of crop growth and development.

(3) The relatively small performance sensitivity to changes in the heat and mass capacities of the greenhouse air indicate that the response time of the greenhouse climate is not a limiting factor for economic optimal control.

(4) The outside climate conditions such as solar radiation, carbon dioxide concentration and humidity, and to a lesser extent the temperature, are important in greenhouse climate management. Consequently their accurate measurement and prediction is required.

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